Program : **B.Tech**

Subject Name: **Structural Analysis-II**

Subject Code: **CE-503**

Semester: **5th**

UNIT-II

Plastic analysis of beams and frames.

Basics and Principles of Plastic Analysis

Plastic analysis is defined as the analysis in which the criterion for the design of structures is the ultimate load. We can define it as the analysis inelastic material is studied beyond the elastic limit (which can be observed in stress strain diagram). Plastic analysis derives from a simple mode failure in which plastic hinges form. Actually the ultimate load is found from the strength of steel in plastic range. This method of analysis is quite rapid and has rational approach for analysis of structure. It controls the economy regarding to weight of steel since the sections required by this method are smaller than those required by the method of elastic analysis. Plastic analysis has its application in the analysis and design of indeterminate structures.

Basics of Plastic analysis:

Plastic analysis is usually based on the idealization of stress strain curve as perfectly plastic. In this analysis it is assumed that width thickness ratio of plate elements is small so the local buckling does not occur. Broadly speaking the section will be declared as perfectly plastic. Keeping in mind these assumptions, it can be said that section will reach its plastic moment capacity and after that will be subjected to considerable moment at applied moments.

Principles of Plastic analysis:

There are following conditions for plastic analysis

1. Mechanism condition
2. Equilibrium condition
3. Plastic moment condition

Mechanism condition:

When the ultimate load is reached collapse mechanism usually formed.

Equilibrium condition:

$\Sigma FX=0$, $\Sigma FY=0$, $\Sigma Mxy=0$

Plastic moment condition:

The bending moment at any section in the structure should not be more than the full plastic moment (moment at which plastic hinges form and structure moves to failure) of the section.

Plastic moment:

If we consider the case of simply supported beam, when the load is gradually applied on it, bending moment and stresses increases. As the load is increased, the stresses in fibers of beam reach to yield stress. At this stage the moment which has converted the stresses into the yield stress is said to be as Plastic moment. it is usually denoted by Mp at this stage the beam member cannot take up any additional moment but may maintain this moment for some amount of rotation and acts like a plastic hinge(hinge means having no capacity to resist moment). Plastic hinge behaves like an ordinary hinge allowing free rotation about itself. The yield moment and plastic moment has relationship which can be described by help of following relation:

My = 2/3 Mp

In calculation of plastic moments the term shape factor has its own importance. Shape factor can be defined as the ratio of plastic moment to yield moment is said to be as the shape factor. Shape factor depend usually on shape of the cross section.

For rectangular cross section the plastic moment can be calculated as:

Yield stress = $(bh^2/4)$

When the load is applied on the body which is elastic (return to its shape after the load is removed), it will show resistance against deformation, such a body is called to be as structure. On the other hand if no resistance is shown against the body, then it is known as mechanism. When plastic hinges equal to n+1 form in the structure, then the structure will collapse (where n is degree of indeterminacy of structure). It means if the plastic hinges in structures increases in number than their degree of indeterminacy, structures move towards collapse.

Plastic hinge and degree of indeterminacy:

Whenever plastic hinge forms in the structure, equilibrium is obtained. As the result the degree of static indeterminacy reduces by one with the formation of one plastic hinge. We can say that if the structure has 'n' number of degree of indeterminacy, its degree of indeterminacy reduces and it becomes determinate structure if 'n' number of plastic hinges forms in it.

Ductility of Steel

Structural steel is characterized by its capacity to withstand considerable deformation beyond first yield, without fracture. During the process of 'yielding' the steel deforms under a constant and uniform stress known as 'yield stress'. This property of steel, known as ductility, is utilized in plastic design methods. Fig.1 shows the idealized stress-strain relationship for structural mild steel when it is subjected to direct tension. Elastic straining of the material is represented by line OA. AB represents yielding of the material when the stress remains constant, and is equal to the yield stress, f_y. The strain occurring in the material during yielding remains after the load has been removed and is called the plastic strain and this strain is at least ten times as large as the elastic strain, H_y at yield point.

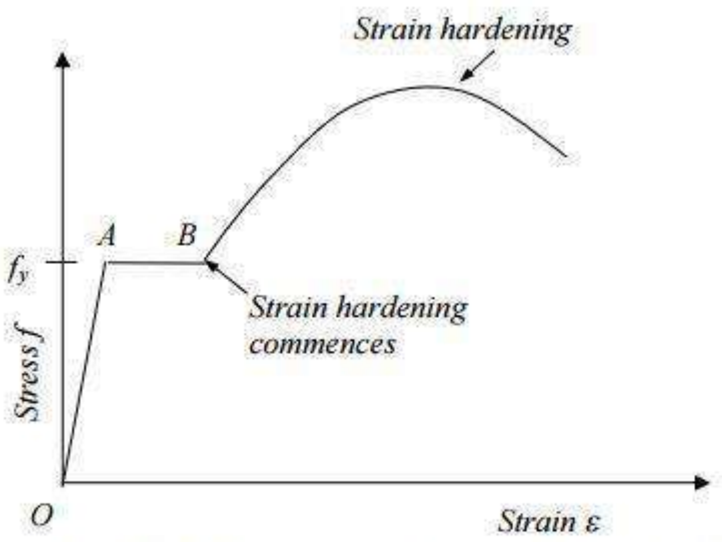


Fig. 1 Idealised stress – strain curve for steel in tension

GENERAL REQUIREMENTS FOR UTILISING PLASTIC DESIGN CONCEPTS

Generally codes (such as IS 800, BS 5950) allow the use of plastic design only where loading is predominantly static and fatigue is not a design criterion.

For example, in order to allow this high level of strain, BS 5950 prescribes the following restrictions on the properties of the stress-strain curve for steels used in plastically designed structures.

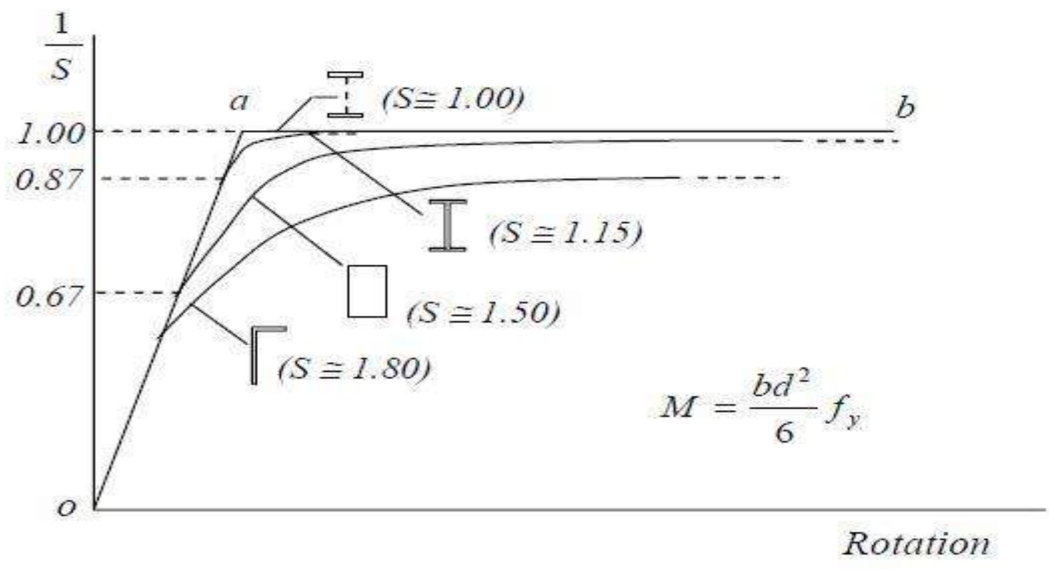
1. The yield plateau (horizontal portion of the curve) is greater than 6 times the yield strain.
2. The ultimate tensile strength must be more than 1.2 times the yield strength.
3. The elongation on a standard gauge length is not less than 15%.

These limitations are intended to ensure that there is a sufficiently long plastic plateau to enable a hinge to form and that the steel will not experience a premature strain hardening.

Shape Factor

As described previously there will be two stress blocks, one in tension, the other in compression, both of which will be at yield stress. For equilibrium of the cross section, the areas in compression and tension must be equal. For a rectangular cross section, the elastic moment.

Here the plastic moment Mp is about 1.5 times greater than the elastic moment capacity. In developing this moment, there is a large straining in the extreme fibers together with large rotations and deflection. This behavior may be plotted as a moment-rotation curve. Curves for various cross sections. The ratio of the plastic modulus (Zp) to the elastic modulus (Z) is known as the shape factor (S) and will govern the point in the moment-rotation curve when non-linearity starts. For the theoretically ideal section in bending i.e. two flange plates connected by a web of insignificant thickness, this will have a value of 1. When the material at the centre of the section is increased, the value of S increases. For a universal beam the value is about 1.15 increasing to 1.5 for rectangle.



FUNDAMENTAL CONDITIONS FOR PLASTIC ANALYSIS

(i) **Mechanism condition:** The ultimate or collapse load is reached when a mechanism is formed. The number of plastic hinges developed should be just sufficient to form a mechanism.

(ii) **Equilibrium condition**: ∑Fx = 0, ∑Fy = 0, ∑Mxy = 0

iii) **Plastic moment condition:** The bending moment at any section of the structure should not be more than the fully plastic moment of the section.

LOAD FACTOR AND THEOREMS OF PLASTIC COLLAPSE

Plastic analysis of structures is governed by three theorems, which are detailed in this section.

The load factor at rigid plastic collapse (□p) is defined as the lowest multiple of the design loads which will cause the whole structure, or any part of it to become a mechanism. In a limit state approach, the designer is seeking to ensure that at the appropriate factored loads the structure will not fail. Thus the rigid plastic load factor □p must not be less than unity.

The number of independent mechanisms (n) is related to the number of possible plastic hinge locations (h) and the number of degree of redundancy (r) of the frame by the equation.

$n = h - r$

The three theorems of plastic collapse are given below for reference.

Mechanism

When a system of loads is applied to an elastic body, it will deform and will show a resistance against deformation. Such a body is known as a structure. On the other hand if no resistance is set up against deformation in the body, then it is known as a mechanism.

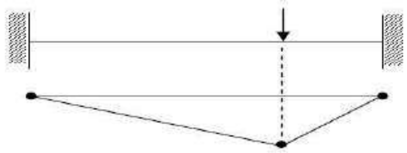
Various types of independent mechanisms are

Beam Mechanism

A fixed beam requires three hinges to Form a mechanism.

Redundancy, r = 2

No. of plastic hinges = 2 + 1 = 3



Panel or Sway Mechanism

Fig.(A) shows a panel or sway mechanism for a portal frame fixed at both ends.

(A) Panel Mechanism (B) Gable Mechanism (C) Joint Mechanism

Gable Mechanism

Fig. (B) Shows the gable mechanism for a gable structure fixed at both the supports.

Joint Mechanism

Fig. (C) Shows a joint mechanism. It occurs at a joint where more than two structural Members meet.

Upper Bound or Kinematic Theorem

A load factor (λk) computed on the basis of an arbitrarily assumed mechanism will always be greater than, or at best equal to the load factor at rigid plastic collapse (λp) λp is the lowest value of λk which can be found.

Uniqueness Theorem

If both the above criteria are satisfied, then the resulting load factor corresponds to its value at rigid plastic collapse (λp).

Stability

For plastically designed frames three stability criteria have to be considered for ensuring the safety of the frame. These are

1. General Frame Stability.
2. Local Buckling Criterion.
3. Restraints.

Effect of axial load and shear

If a member is subjected to the combined action of bending moment and axial force, the plastic moment capacity will be reduced.

Plastic analysis for more than one condition of loading

When more than one condition of loading can be applied to a beam or structure, it may not always be obvious which is critical. It is necessary then to perform separate calculations, one for each loading condition, the section being determined by the solution requiring the largest plastic moment.

Unlike the elastic method of design in which moments produced by different loading systems can be added together, plastic moments obtained by different loading systems cannot be combined, i.e. the plastic moment calculated for a given set of loads is only valid for that loading condition. This is because the 'Principle of Superposition' becomes invalid when parts of the structure have yielded.